Decoding of Polar Codes with Reinforcement Learning

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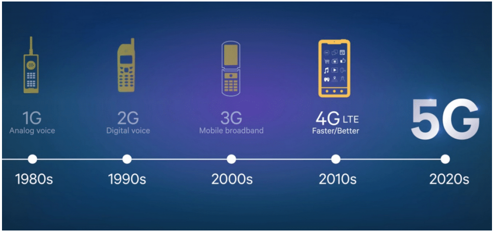
Background



- Polar codes: selected for the eMBB control channel in 5G
- Cyclic redundancy check (CRC) is concatenated with polar codes in 5G for error detection
- Belief Propagation (BP): reasonable error-correction performance, **highly parallel**

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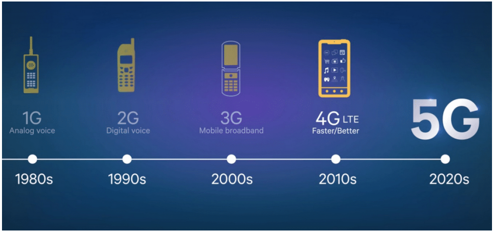

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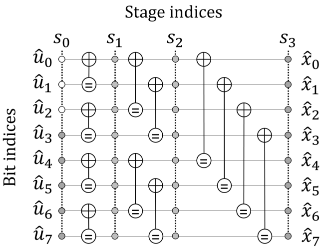
Polar codes

- Introduced by Arıkan in 2009
- $\mathcal{P}(N, K)$, N: code length, K: message length
- Code construction: based on polarization phenomenon
  - K most reliable channels: information bits
  - $(N-K)$ least reliable channels: frozen bits

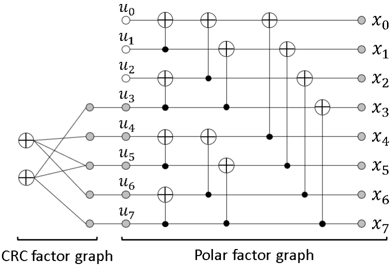


$\mathcal{P}(8, 5)$ with u_0, u_1, and u_2 are frozen bits

E. Arıkan, "Channel Polarization: A Method for Constructing Capacity-Achieving Codes for Symmetric Binary-Input Memoryless Channels", IEEE Trans. on Info. Theory, vol. 55, no. 7, pp. 30513073, July 2009.

CRC-Aided BP (CABP) Decoder of Polar Codes

- Exploit the extrinsic information of the CRC-polar factor graphs
  - Perform BP decoding on the polar factor graph I_{th} iterations
  - Unsatisfied CRC test: run BP decoding on the CRC graph



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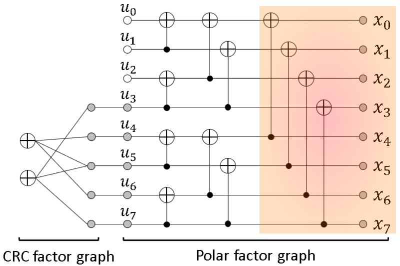

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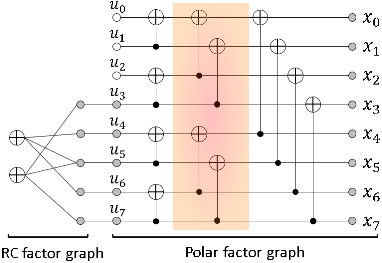

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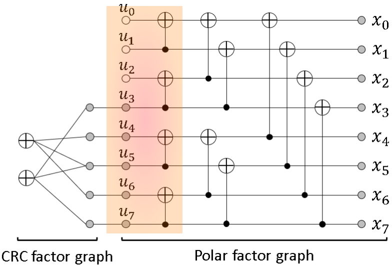

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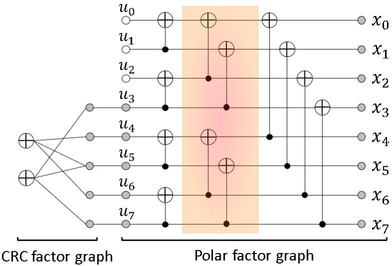

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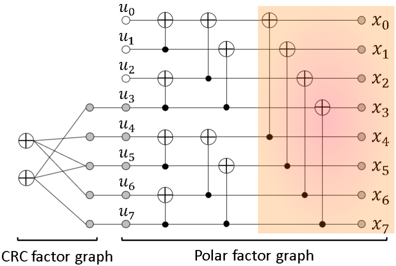

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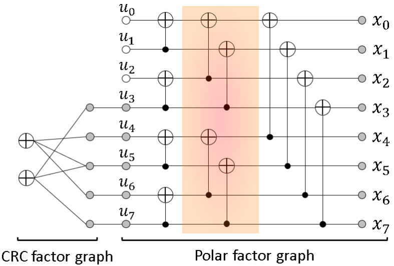

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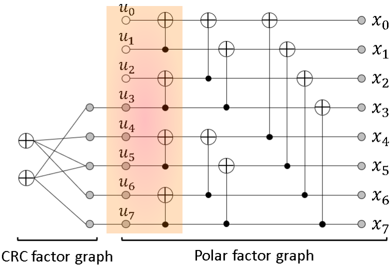

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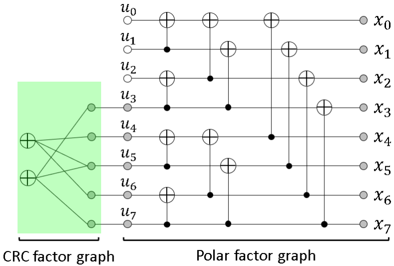

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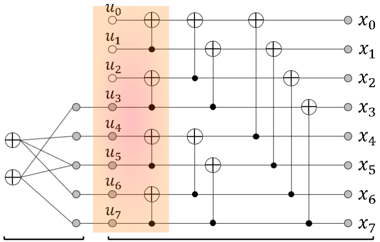

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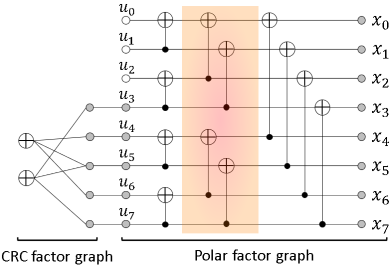

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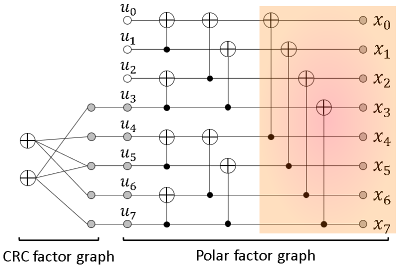

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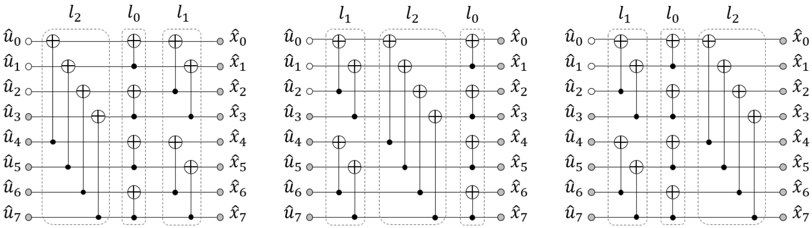
Polar Decoding with Permuted Factor Graphs

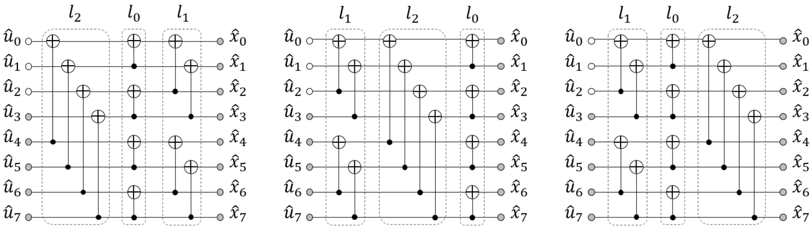
- Permuting the PE layers of the polar code factor graph does not change the code
- The error probability of BP decoding can be improved by using different factor graph permutations



Random factor-graph permutations of polar codes

Polar Decoding with Permuted Factor Graphs

- Permuting the PE layers of the polar code factor graph does not change the code
- The error probability of BP decoding can be improved by using different factor graph permutations
- Open problem: Under a specific channel output, select a factor graph that results in a correct codeword.



Random factor-graph permutations of polar codes

Previous Work

- Cyclic factor-graph permutations [a]
- Random factor-graph permutations [b]
- Monte Carlo based methods [c-e]

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- Cyclic factor-graph permutations [a]
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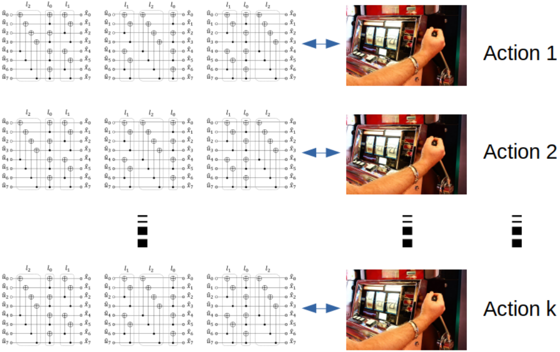
- Given a channel output $\boldsymbol{y}$, select a set of T ”good” factor-graph permutations for CABP during the decoding
- Formalize the factor-graph permutation selection as a multi-armed bandit problem $\rightarrow$ use state-of-the-art bandit algorithms to solve the problem

Decoding of Polar Codes with Reinforcement Learning



- Generate *k* random actions (sets of permutations)
- Each action contains the original permutation and a set of $T - 1$ random permutations
- The decoder selects a set of permutations (an action) to perform CABP decoding

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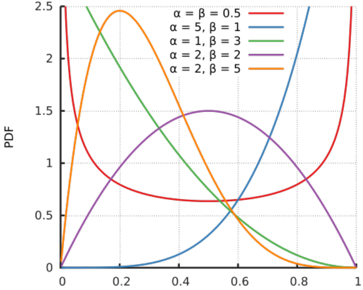

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- Action selection:

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- Action selection:
  - $\epsilon - \textit{Greedy}$

$$a^* = \begin{cases} \arg\max_{\forall a_j} v_{a_j} & \text{with prob. } 1 - \epsilon \text{ (exploitation)} \\ a_{\text{random}} & \text{with prob. } \epsilon \text{ (exploration)} \end{cases}$$

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  - Upper Confidence Bound (UCB)

$$a^* = \arg\max_{\forall a_j} \left[\underbrace{v_{a_j}}_{\text{exploitation}} + \underbrace{c\sqrt{\frac{\ln t}{n_{a_j}}}}_{\text{exploration}} \right]$$

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- Action selection:
  - Thompson Sampling (TS)

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An action k is then selected as $a^* = \arg\max_{\forall j} v_{a_j}$

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    $$v_{a_j} = \text{BetaDist}(\alpha_j, \beta_j)$$

    An action k is then selected as $a^* = \arg\max_{\forall j} v_{a_j}$
- *The action selection can be obtained prior to the actual decoding.*

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$$v_{a^*} := v_{a^*} + \frac{\mathbb{1}_{E_{a^*}} - v_{a^*}}{n_{a^*}}$$

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$$\alpha_k := \alpha_k + 1 - \mathbb{1}_{E_{a_k}}$$
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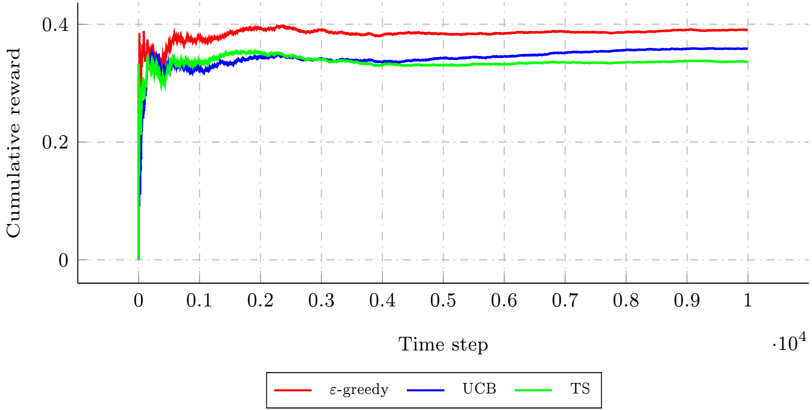

Performance comparison of various multi-armed bandit algorithms used by RL-CABP decoding. The simulation is obtained at $E_b/N_0 = 3.0$ dB with $k = 500$, $\varepsilon = 2^{-4}$, and $c = 2^{-3}$.

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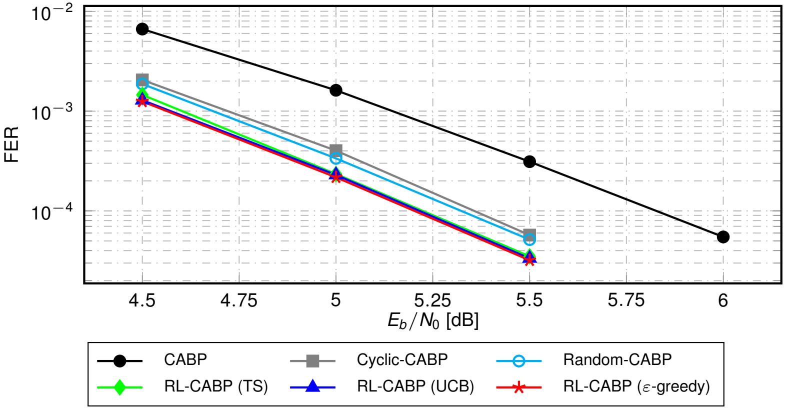

Error-correction performance of different factor-graph permutation selection schemes for $\mathcal{P}(128, 64)$ with a 24-bit CRC used in 5G (24C).

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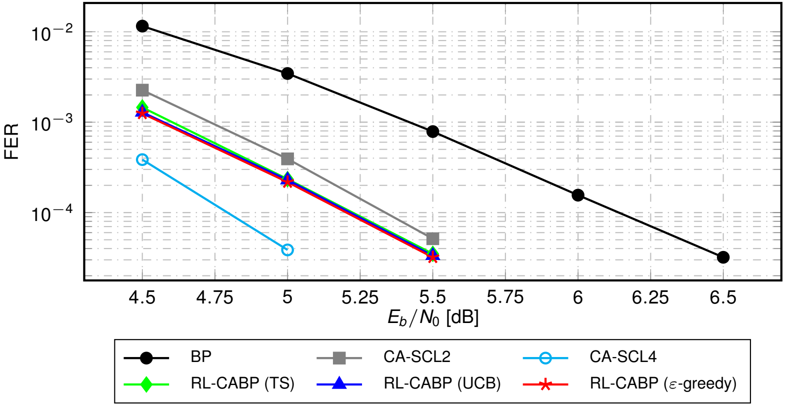

Error-correction performance of RL-CABP decoding and other decoding algorithms of polar codes.

Decoding of Polar Codes with Reinforcement Learning

- Propose an algorithm that selects the good factor-graph permutations during the course of decoding
- Significantly reduce the error-probability of BP decoding while still maintain the parallelism property of BP decoding
- The factor-graph selection can be pipelined with the decoding process

Thank You!