Neural Belief Propagation Decoding of CRC-Polar Concatenated Codes

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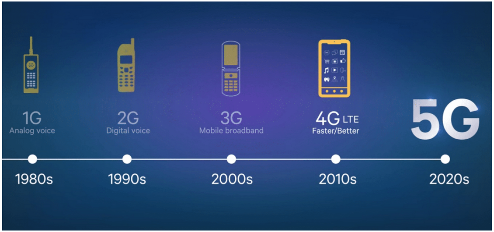
May 22, 2019

Background



- Polar codes: selected for the eMBB control channel in 5G
- Cyclic redundancy check (CRC) is concatenated with polar codes in 5G for error detection
- Belief Propagation (BP): reasonable error-correction performance, **highly parallel**

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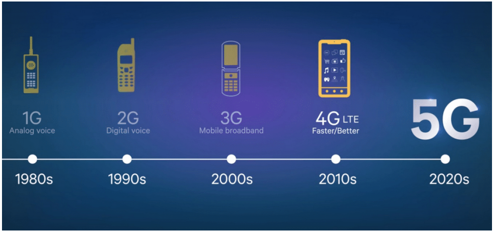

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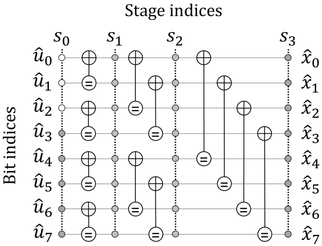
- Cyclic redundancy check (CRC) is concatenated with polar codes in 5G for error detection
- Belief Propagation (BP): reasonable error-correction performance, **highly parallel** → **high decoding throughput**

Contribution

- Exploit the inherent CRC-polar concatenated factor graph to improve the error-correction performance under BP decoding
- Assign trainable weights to the concatenated factor graph to reduce the number of decoding iterations

Polar codes

- Introduced by Arıkan [1] in 2009
- $\mathcal{P}(N, K)$, N: code length, K: message length
- Code construction: based on polarization phenomenon
  - K most reliable channels: information bits
  - $(N - K)$ least reliable channels: frozen bits



$\mathcal{P}(8, 5)$ with u_0, u_1, and u_2 are frozen bits

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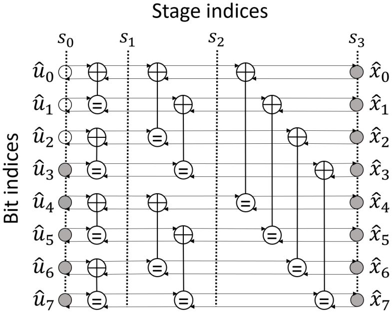
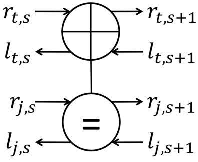
[1] E. Arıkan, "Channel Polarization: A Method for Constructing Capacity-Achieving Codes for Symmetric Binary-Input Memoryless Channels", IEEE Trans. on Info. Theory, vol. 55, no. 7, pp. 3051–3073, July 2009.

Belief Propagation (BP) Decoding

- Iterative message-passing algorithm
- Termination: CRC-based with predefined $I_{\max}$ iterations
- Messages are calculated by Processing Elements (PEs)
- The message-passing operations can be unfolded



BP decoding for $\mathcal{P}(8,5)$



A polar PE

l: right-to-left messages
r: left-to-right messages
$0 \leq t, j \leq N-1$: bit indices
$0 \leq s \leq \log_2(N)$: stage indices

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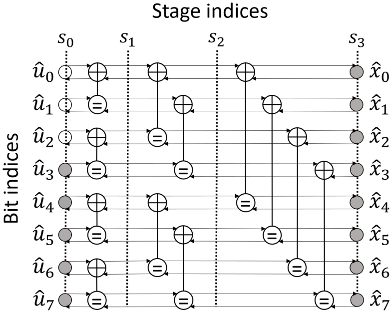
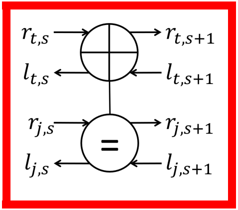


BP decoding for $\mathcal{P}(8, 5)$

Unfold



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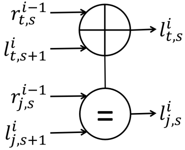
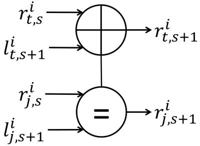


A right-to-left polar PE

$$\begin{cases} l_{t,s}^{i} &= f(l_{t,s+1}^{i}, r_{j,s}^{i-1} + l_{j,s+1}^{i}) \\ l_{j,s}^{i} &= f(l_{t,s+1}^{i}, r_{t,s}^{i-1}) + l_{j,s+1}^{i} \end{cases}$$



A left-to-right polar PE

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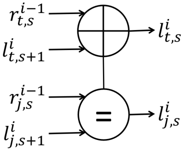
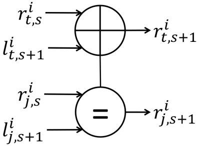
i: iteration index, $f(a,b) = \min(|a|,|b|)\,\mathrm{sgn}(a)\,\mathrm{sgn}(b)$

Neural BP Decoding [2,3]



$$\begin{cases} l_{t,s}^{i} &= w_0 f(l_{t,s+1}^{i}, w_1 r_{j,s}^{i-1} + w_2 l_{j,s+1}^{i}) \\ l_{j,s}^{i} &= w_4 (w_3 f(l_{t,s+1}^{i}, r_{t,s}^{i-1})) + w_5 l_{j,s+1}^{i} \end{cases}$$

A right-to-left polar PE



$$\begin{cases} r_{t,s+1}^{i} &= w_6 f(r_{t,s}^{i}, w_7 l_{j,s+1}^{i} + w_8 r_{j,s}^{i}) \\ r_{j,s+1}^{i} &= w_{10} (w_9 f(r_{t,s}^{i}, l_{t,s+1}^{i})) + w_{11} r_{j,s}^{i} \end{cases}$$

A left-to-right polar PE

$w_m \in \mathbb{R}^+$ $(0 \leq m \leq 11)$: trainable weights

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[3] W. Xu et al., “Improved polar decoder based on deep learning,” in IEEE Int. Workshop on Signal Process. Syst., November 2017, pp. 1–6.

CRC-Polar BP (CPBP) Decoding

- Exploit the CRC-Polar concatenated factor graph
- Run BP decoding on the CRC factor graph after I_{thr} iterations
- The choice of I_{thr} affects the error-correction performance

Algorithm: **CPBP-**$(\boldsymbol{I}_{\text{max}}, \boldsymbol{I}_{\text{thr}})$

```
for $i = 0$ to $I_{max}$ do
    Polar BP Decoding
    if $i > I_{thr}$ then
        CRC BP Decoding
    if $CRC == True$ then
        Output and Terminate
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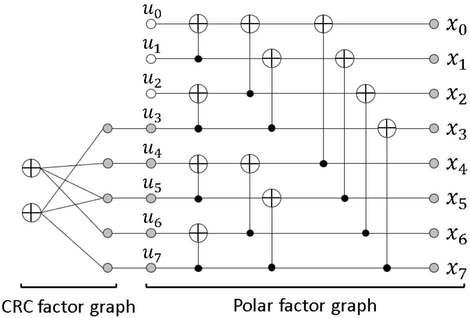

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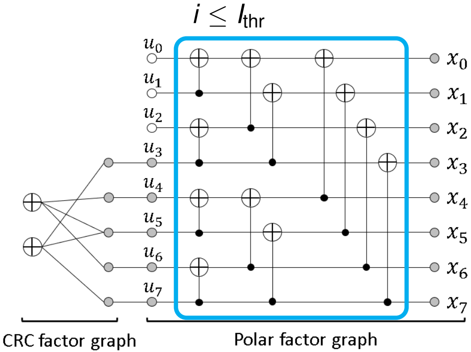

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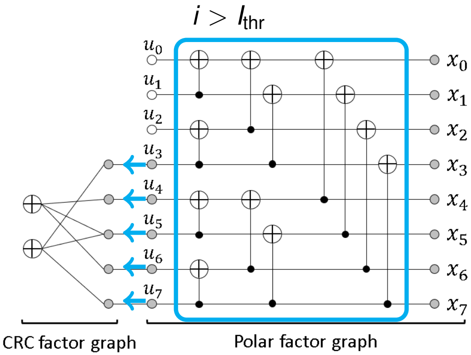

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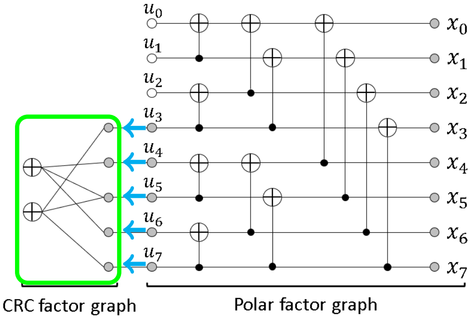

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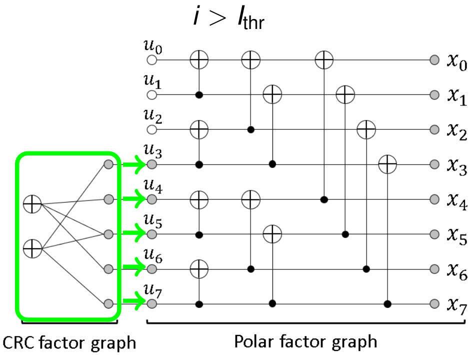

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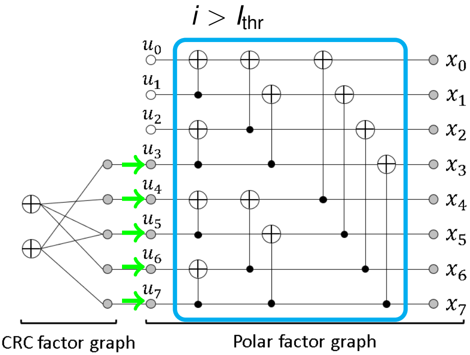

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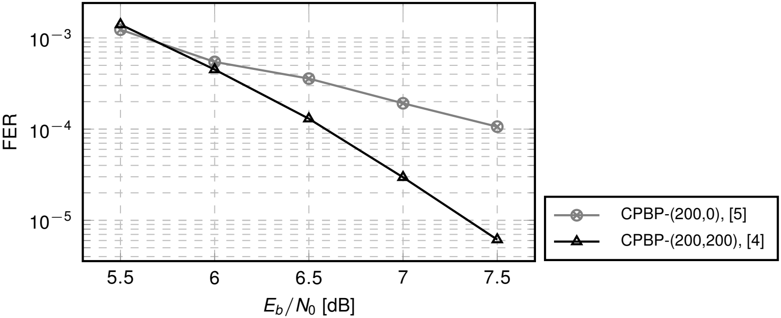

FER performance of various CPBP-($I_{\max}, I_{\text{thr}}$) decoders for $\mathcal{P}(128, 80)$ and a 16-bit CRC used in 5G.

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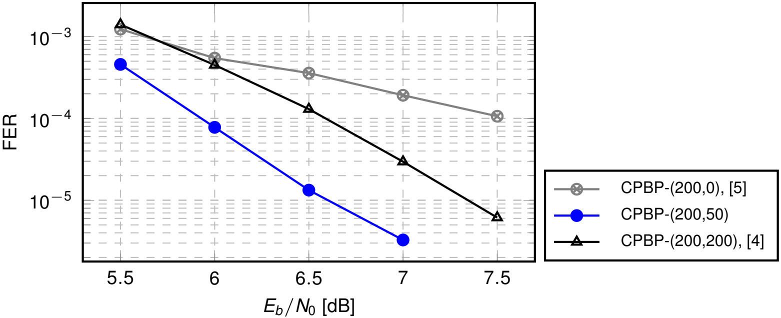

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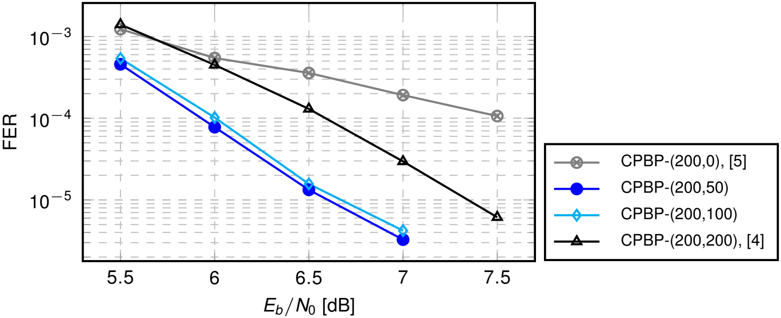

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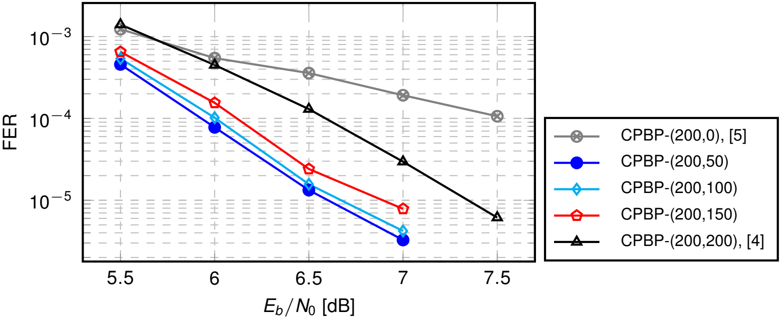

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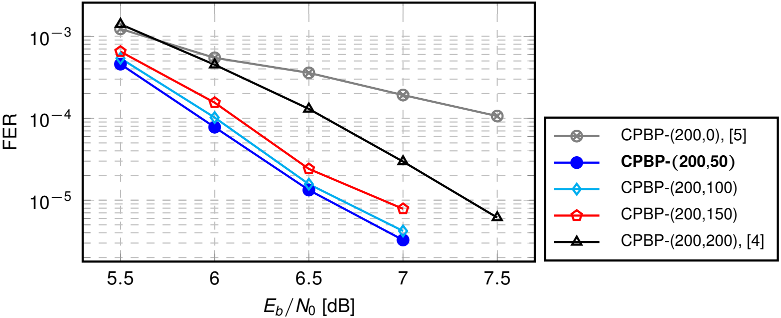

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- A small value of $I_{\max}$ is needed for applications with strict latency requirements

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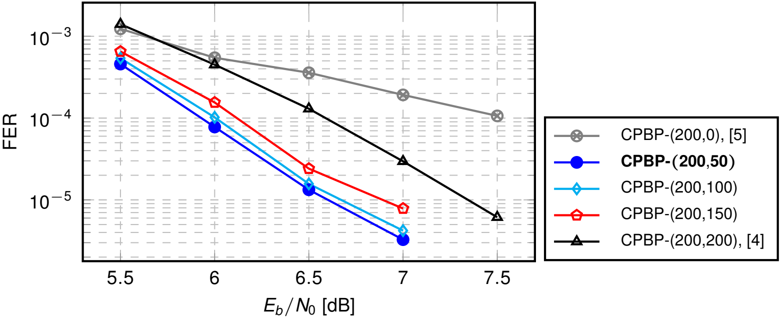

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- A small value of $I_{\max}$ is needed for applications with strict latency requirements → **assign trainable weights to reduce $I_{\max}$**

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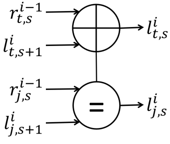
Neural CRC-Polar BP (NCPBP) Decoding

- Revise efficient weight assignment and sharing schemes for the CPBP decoder
- Preserve the symmetric property of the conventional BP decoding algorithm
- Training using all-zero codewords with stochastic gradient-descent based techniques

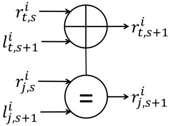
Neural CRC-Polar BP (NCPBP) Decoding

Weight assignment scheme



$$\begin{cases} l_{t,s}^{i} &= w_0 f(l_{t,s+1}^{i}, w_{1,2}(r_{j,s}^{i-1} + l_{j,s+1}^{i})) \\ l_{j,s}^{i} &= w_{3,4} f(l_{t,s+1}^{i}, r_{t,s}^{i-1}) + w_5 l_{j,s+1}^{i} \end{cases}$$

A right-to-left polar PE



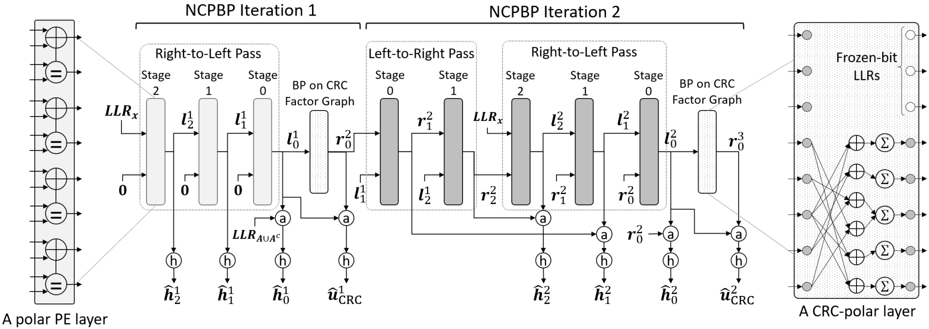
$$\begin{cases} r_{t,s+1}^{i} &= w_6 f(r_{t,s}^{i}, w_{7,8}(l_{j,s+1}^{i} + r_{j,s}^{i})) \\ r_{j,s+1}^{i} &= w_{9,10} f(r_{t,s}^{i}, l_{t,s+1}^{i}) + w_{11} r_{j,s}^{i} \end{cases}$$

A left-to-right polar PE

$w_m \in \mathbb{R}^+$ are trainable weights.

Neural CRC-Polar BP (NCPBP) Decoding

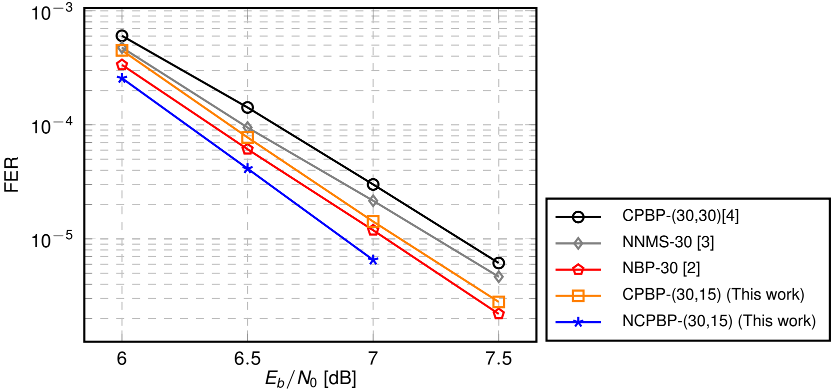
Weight sharing scheme



$$\mathbf{x},\mathbf{y} \rightarrow \textcircled{a} \rightarrow \mathbf{x}+\mathbf{y} \qquad \mathbf{x} \rightarrow \textcircled{h} \rightarrow \begin{cases} \dfrac{1}{1+e^{\mathbf{x}}}, & \text{when training,} \\ \dfrac{1-\operatorname{sgn}(\mathbf{x})}{2}, & \text{when testing.} \end{cases} \qquad \mathbf{x},\mathbf{y}\in\mathbb{R}^N$$

- All polar PE layers in an iteration share the same weights
- All CRC-polar layers share the same set of weights

Neural CRC-Polar BP (NCPBP) Decoding



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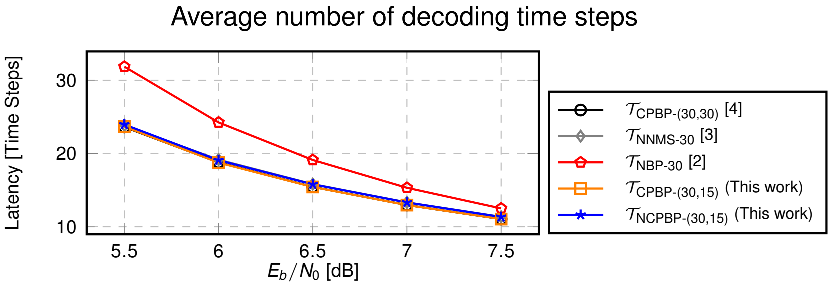

- CRC-based early termination BP-based decoding:

$$\mathcal{T}_{\text{BP}} = (2n-1)(I_{\text{ET}}-1)+n$$

- Neural CPBP decoding:

$$\mathcal{T}_{\text{CPBP}} = \begin{cases} (2n-1)(I_{\text{ET}}-1)+n, & \text{if } I_{\text{ET}} \leq I_{\text{thr}} \\ (2n-1)(I_{\text{ET}}-1)+n+2(I_{\text{ET}}-I_{\text{thr}}) & \text{otherwise.} \end{cases}$$

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Neural CRC-Polar BP (NCPBP) Decoding

Number of weights required by different neural BP decoders.

Decoder	Number of weights
NBP-30 [2]	11520
NCPBP-(30,15) (This work)	8288
NNMS-30 [3]	3840

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Conclusion

- We proposed CRC-polar BP (CPBP) decoding
- We proposed Neural CRC-polar BP (NCPBP) decoding with efficient weight assignment and sharing schemes
- For a 5G $\mathcal{P}(128, 80)$ concatenated with a 16-bit CRC, NCPBP achieves up to 0.4 dB performance gain compared to state of the art, with almost no latency overhead

Thank You!