Fast SC-Flip Decoding of Polar Codes with Reinforcement Learning

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Preliminaries & Problem Statements
Background

- Polar codes: selected for the eMBB control channel in 5G
- Cyclic redundancy check (CRC) is concatenated with polar codes in 5G for error detection
- CRC-polar concatenated codes are decoded using Successive-Cancellation (SC) based decoding algorithms
Polar codes

- Introduced by Arıkan in 2009 [Arıkan’09]
- $\mathcal{P}(N, K)$, $N$: code length, $K$: message length
- $\mathcal{A}$: information-bit set, $|\mathcal{A}| = K$, contains the reliable channels
- $\mathcal{A}^c$: frozen-bit set, $|\mathcal{A}^c| = N - K$, contains the noisy channels

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SC and Fast-SC Decoding

- SC decoding traverses all the nodes under the binary-tree representation of the code, rendering a high decoding latency.
- Fast-SC decoding performs the decoding at the parent-node level of some special nodes → reduce the decoding latency [Sarkis’14]

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SC-Flip (SCF) Decoding

- Given that the first SC decoding is not successful
- Estimation of the first error bit: $i_e^* = \arg \min_{\forall i \in A} |\alpha_{0,i}|$
- E.g., $i_e^* = 6$, thus $\hat{u}_6$ is flipped from 1 to 0 in the second SC decoding attempt

Dynamic SC-Flip (DSCF) Decoding

- Estimation of the first error bit: \( i^*_e = \arg\min_{i \in A} Q_i \), where

\[
Q_i = |\alpha_{0,i}| + \sum_{\forall j \in A, j \leq i} \frac{1}{\delta} \ln [1 + \exp (-\delta |\alpha_{0,j}|)],
\]

and \( \delta = 0.3 \) is a perturbation parameter.

- E.g., \( i^*_e = 5 \), thus \( \hat{u}_1 \) is flipped in the second SC decoding attempt

<table>
<thead>
<tr>
<th>( i \in A )</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>(</td>
<td>\alpha_{0,i}</td>
<td>)</td>
<td>1.1</td>
<td>0.7</td>
<td>1.9</td>
<td>2.4</td>
<td>3.8</td>
<td>2.2</td>
</tr>
<tr>
<td>( Q_i )</td>
<td>2.9</td>
<td>4.5</td>
<td>7.2</td>
<td>9.0</td>
<td>11.3</td>
<td>11.1</td>
<td>12.1</td>
<td>16.3</td>
</tr>
</tbody>
</table>

- DSCF requires costly computations and the decoding is performed at the leaf-node level, which results in a high decoding latency.

Contributions

- Propose a novel bit-flipping algorithm tailored to Fast-SC decoding.
- Use a parameterized model, which is optimized using reinforcement learning (RL).
- Has a similar or better error-bit prediction accuracy when compared with the state-of-the-art SCF-based algorithms.
The Proposed Algorithm
Construction of the LLR vector $\gamma$

- $\gamma$: a vector of LLR values of the visited nodes under Fast-SC decoding.
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- $\nu$: SPC node
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- Parity check condition:
  \[ 0 = \bigoplus_{i = i_{\min \nu}}^{i_{\max \nu}} \beta_{s,i} \rightarrow \beta_{s,i_{\min \nu}} = \bigoplus_{i = i_{\min \nu} + 1}^{i_{\max \nu}} \beta_{s,i} \rightarrow \text{only flip } \beta_{s,i} \]
  where $i_{\min \nu} < i \leq i_{\max \nu}$
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    \[ 0 = \bigoplus_{i = i_{\min}}^{i_{\max}} \beta_s, i \rightarrow \beta_s, i_{\min} = \bigoplus_{i = i_{\min} + 1}^{i_{\max}} \beta_s, i \rightarrow \text{only flip } \beta_s, i \]
    where $i_{\min} < i \leq i_{\max}$
  - Update $\gamma \leftarrow \gamma \cup \alpha_{s, i}$ for all $i_{\min} < i \leq i_{\max}$
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    \]
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\[
\gamma = \{\alpha_{2,5}, \alpha_{2,6}, \alpha_{2,7}\}
\]
Construction of the LLR vector $\gamma$

- $\nu$: REP node
  - Update $\gamma \leftarrow \gamma \cup \alpha_{0,i_{\text{max}}}, \text{ where } \alpha_{0,i_{\text{max}}} = \sum_{i=i_{\text{min}}}^{i_{\text{max}}} \alpha_{s,i}$

\[ \gamma = \{ \alpha_{2,5}, \alpha_{2,6}, \alpha_{2,7}, \alpha_{0,11} = \sum_{i=8}^{11} \alpha_{2,i} \} \]
Construction of the LLR vector $\gamma$

- $\nu$: Rate-1 node
  - Update $\gamma \leftarrow \gamma \cup \alpha_i$ for all $i_{\min,\nu} \leq i \leq i_{\max,\nu}$

- If the $i$-th bit of $\gamma$ is an error bit $\rightarrow$ flip the hard decision corresponding to the $i$-th bit of $\gamma$
The error bit is estimated by learning the correlation of the elements of $\gamma$. 

Let $\theta$ be a $(K + C) \times (K + C)$ correlation matrix where $\theta_{i,i} = 1$ and $\theta_{i,j} = \theta_{j,i}$ for $0 \leq i, j < K + C$.

The bit-error metric $M_i$ of the $i$-th element of $\gamma$: 

$$M_i = \sum_{0 \leq j < K + C} \theta_{i,j} |\gamma_j|.$$ 

The most probable error index: $i^* = \arg \min_{0 \leq i < K + C} M_i$. 

Estimation of The Error Position
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  \] (1)
- The most probable error index: $i_e^* = \arg \min_{0 \leq i < K+C} M_i$. 
Optimization of $\theta$ using a RL setup

FSC decoding with bit-flipping operations  
\[ y \rightarrow \text{FSC decoding with bit-flipping operations} \rightarrow r^* \rightarrow \text{Bit-flipping policy $\pi_{\theta}$} \]

Environment  
Agent

- If flipping the bit associated with the $a^*$-th element of $\gamma$ results in a CRC pass $\rightarrow r^* = 1$, otherwise $r^* = 0$.

- Update $\theta$ using the gradient ascent technique

\[
\theta \leftarrow \theta + \frac{\ln p_{a^*}}{\text{d} \theta} (r^* - \bar{r}), \quad (2)
\]

where $\bar{r}$ is the cumulative average reward.
Performance Evaluation

\[ P(512, 256), \ C = 24, \ # \ of \ flipping \ attemp \ T_{\text{max}} = 1 \]


Performance Evaluation

The FERs of various fast SCF decoding algorithms as a function of $T_{\text{max}}$.

$E_b/N_0 = 3 \, \text{dB}$

$E_b/N_0 = 4 \, \text{dB}$

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Average number of decoding attempts $T_{\text{avg}}$ of various fast SCF decoding algorithms with $T_{\text{max}} = 8$.


Conclusion

- Proposed a novel bit-flipping algorithm for Fast-SC decoding
- Better error-bit estimation accuracy compared to that of state-of-the-art Fast-SCF decoding algorithms, given a small number of flipping-attempts.
- Using RL technique to optimize the parameters, which can be carried out at the decoder side and does not require pilot signals.
Thank You!